Chapter 13

Paradoxes of Relativity

The conceptual complications of the special theory of relativity are often expressed through stories whose outcomes are counter intuitive, paradoxes. The following three are a representative sample.

13.1 The Twin Paradox

13.1.1 The Problem

Alphonse and Gaston are twins and they are authors. Alphonse writes advertising copy and has to travel to town every day and Gaston writes novels and stays home. Each day when Alphonse is on the train going to town he is observed by Gaston. Due to their relative motion, Gaston sees Alphonse's clock running slower and thus Alphonse is aging slower than he does. At the end of the day, when Alphonse has returned home he has not aged as much as Gaston and is therefore younger. The problem is that, during the trip, Alphonse observes Gaston. He notes that Gaston's clock is the one that runs slow. He expects that, when they get back together, Gaston will be younger. When they get back together are they the same age? If there is a difference in their ages, who is younger. The clue to the problem is that Alphonse spills a drink on his shirt every day.

13.1.2 The Solution

Actually, we have already solved this paradox. This is Harry and Sally of Section 11.3.5 on page 265and 12.7 on page 299. The supposed paradox here is that it seems that Alphonse and Gaston are identical. Not only are they twins but they both see the others clock run slow. The simple fact is that they are not identical. This clue is the answer to the paradox; Alphonse spills his drink on his shirt because he is accelerated. Gaston never spills his drink; he is not accelerated. Acceleration is knowable, velocity is not, see Section 9.2 on page 215. Now that we understand that they are no longer identical, one was accelerated, they can be different and it can be that one can now be older than the other one when they get back together. From Section 12.7 on page 299, since the straight line time-like trajectory is the longest, the non-accelerated twin is always the oldest. The exact age difference can be computed from the trajectories of each twin in any convenient frame. The example of Harry and Sally, Section 11.3.5 on page 265, is straight forward.

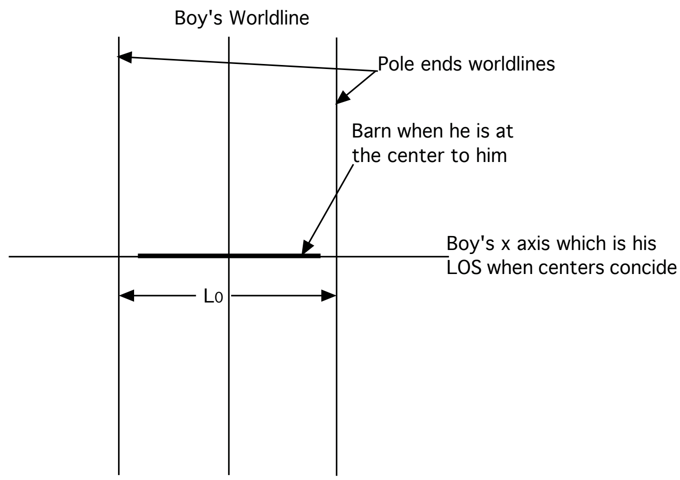
13.2 The Boy in the Barn

13.2.1 The Problem

A boy is a pole vault freak. He runs around a track all day to practice. He also can run very fast, $\frac{3}{5}$c. He has to pass through a barn. In fact, the pole that he practices with is taken from the roof beam of the barn and is the same length as the barn when they are at rest together. He practices all day and his parents worry about him. They want to stop him and make him come in for dinner. He agrees that, if he and his pole are ever entirely in the barn, they can close the front and back doors. His reasoning is that since his pole is much longer than the barn, there is no problem. They will never get him. They agree to do as he says. The clue here is that parents are always right. Do they get him?

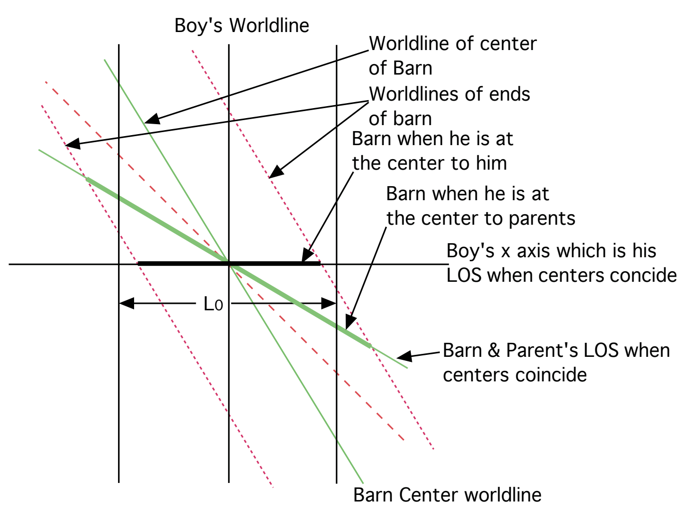
13.2.2 The Solution

This is a classic example of the problem of simultaneity for space-like event intervals, Section 10.3 on page 233. Let's draw a space time diagram from the boy's respective to understand his argument. The rest length of the pole and the barn are L_0. He says that barn length is $\sqrt{1-\left(\frac{3}{5}\right)^2}L_0 = \frac{4}{5}L_0$ which means that if, he measures it when he is at the center of the barn, his assertion is clearly true; both ends of his pole are outside the barn when he is in the center.



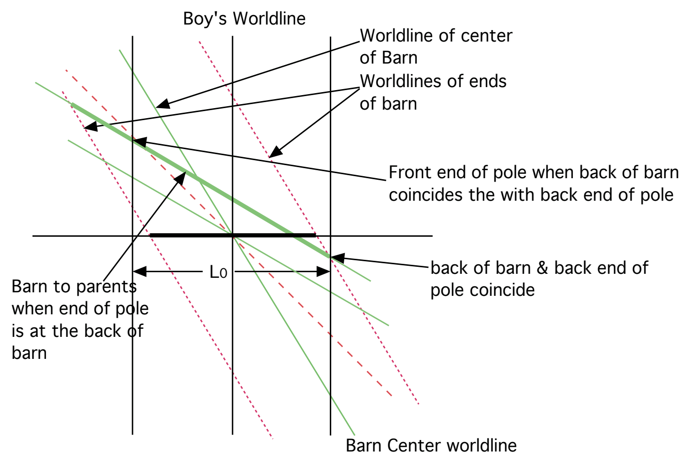
The ends of the pole's worldlines are the thin vertical black lines.

Now, let's add the information about barn's worldlines to this. The center of the barn and boy worldlines meet at the origin. The worldline of the center of the barn is the thin green line sloped at $\frac{5}{3}$ with similar parallel lines with shown in dotted red at the same slope for the ends of the barn.



A light line passing through the origin is dashed red. Also shown by a thin green iine is the LOS for the parents for the event of the coincidence of the center of the pole and the center of the barn and a thick green line between the ends of the barn for the barn at the time of this coincidence for the parents. It is clear the pole is entirely inside the barn to them; two pole end worldlines intersect the barn, the thick green line.

The parents can close the barn doors when the back end of the pole arrives at the back of the barn since the front of the pole is well short of the end of the barn at that time to them.



13.3 The Bandits and the Bullet Train

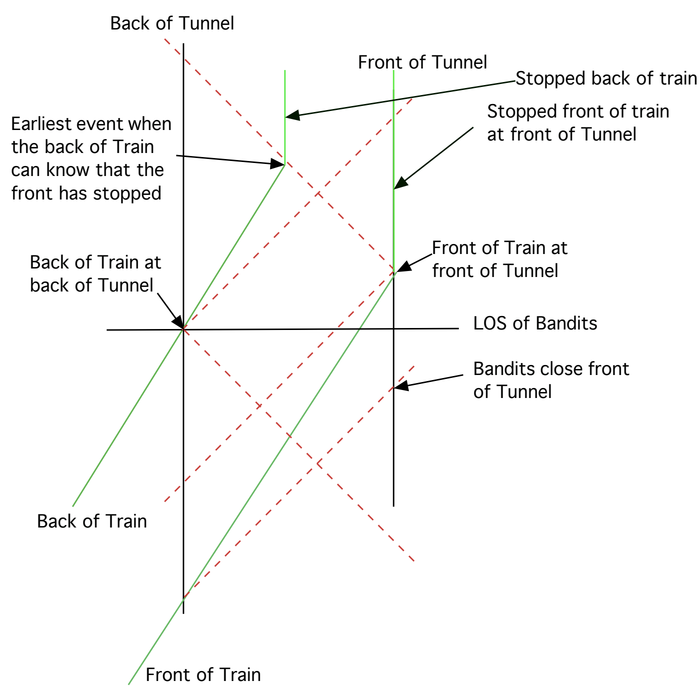
13.3.1 The Problem

The Japanese bullet train travels at $\frac{3}{5}c$. It carries gold bullion from Osaka to Tokyo. It passes through a tunnel that is exactly as long as the train when each are measured at rest. A group of bandits want to capture the train and get the gold. In their frame, the train is shorter than the tunnel and thus can be captured inside of it. The owners of the train know about the tunnel and are concerned about the gold. They decide that, since the train is longer than the tunnel, it will be sufficient to post a guard at the front and the back of the train and thus there will always be one of the guards outside the tunnel. The bandits attempt to capture the gold. They have two ways to accomplish this. One is to rely on the train schedule and close the tunnel gate at the far end from the train right after the the train back enters the tunnel. Another is to wait at the entrance to the tunnel and when the back of the train passes close the back and send a signal to the baddies at the front exit end to close the gate. The train people are not worried about a capture of the train since they can put guards at both ends of the train and there is never a time when the entire train is in the tunnel. Will these plans succeed? If they will, how do the guards on the train describe the situation? Show all of this on a space time diagram.

13.3.2 The Solution

This problem is very similar to *The Boy in the Barn* puzzle above, see Section 13.2 on page 304. That depended on careful analysis of the relativity of simultaneity for space-like event pairs. Here there is, in addition, the issue of signaling between between these kinds of events. Let's treat the two approaches to the problem separately. If the bandits know the train schedule and the train is on schedule this is the same as the *Boy in the Barn* problem. This allows them to close both front and back tunnel gates simultaneously to them when the train is inside.

It is more realistic to do the capture in the other scenario. The bandits wait until the front of the train is at the back of the tunnel. They signal the bandits at the front of the tunnel to close the entrance to the tunnel. Later, because the train is traveling at less than the speed of light, the front of the train arrives at the front of the tunnel. It crashed to a stop. This is after the back of the train is at the back of the tunnel. Thus any guard at the end of the train is captured in the tunnel and there is no chance to send an alarm. Bandits win. All this is shown below.



13.3.3 Consequences*

It is interesting to speculate on the fate of the train. Let's get concrete or, at least, as concrete as makes sense in this context. As usual, we will 'Spherical Cow' the problem. One extreme version of a train is like a linked chain that can only be pulled; In physics language, a continuous system which can only sustain a tension stress. You cannot push on the front end of a chain, you can only pull. Pushing piles the links into a mess. The other is to treat the train as a system of connected cars that are themselves rigid finite lengths. A system of these can sustain a compression or extension stress. This is generally called an elastic solid rod.

Use a rest length for the train as L_0. It's length to the bandits is $\frac{4}{5}L_0$. Using the event of the back of the train at the back of the tunnel as the origin event. The front of the train is at the LOS of the bandits at the event labeled $(\frac{4}{5}L_0, 0)$. This makes the trajectory of the front of the train until it stops at the gate, $ct = \frac{5}{3}(x - \frac{4}{5}L_0)$. The front of the train therefore comes to the front of the tunnel at $x_{front} = L_0$ or $ct_{front} = \frac{5}{3}(1 - \frac{4}{5})L_0 = \frac{1}{3}L_0$. I am assuming the front of the train comes abruptly to a stop at the front of the tunnel. The earliest that the back of the train can know about the stopping event of the front of the train is from a light signal. The trajectory of the light going to the back of the train is $ct = -x + L_0 + \frac{1}{3}L_0$. The trajectory for the back of the train is $\frac{3}{5}ct = x$. The coincident event for these trajectories is $(\frac{1}{2}L_0, \frac{5}{6}L_0)$ and we anticipate abrupt stopping. To the bandits at the time the back of the train stops,it is only half as long as it was and now it is comoving with them. How do you get a train to shrink to half its size? Two answers depending on what you think a train is. If it is a chain, it is a large mess of links. The inertia of the links cause them to pile on top of each other or crush the links. If it is an elastic rod, the rod would experience a compression wave from the abrupt stopping at the front. This wave would propagate back along the rod at the speed of sound. The speed of sound in solids is much lower than the speed of light. In fact each part of the train is forced to stop faster than this process allows. This is what is called a shock wave. The shock wave front creates high temperatures and will actually leave the rod as a pool of molten stuff; another way to say a mess. Another way to say this is that interatomic forces that were responsible for us to call it a solid or a set of links cannot do the job of maintaining the properties that we ascribe to the train. Of course, all of this assumes the gate at the front of tunnel can stop the train. It is clearly a very strong gate.